

The chiral limit of QCD and above

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I review aspects of chiral dynamics pertinent to the structure of baryons and few-nucleon systems, such as chiral extrapolations for the nucleon and the delta mass, double pion photoproduction off protons, single neutral pion electroproduction off the deuteron, pion photoproduction in the delta region, the quark mass dependence of the nuclear forces and the possibility of an infrared limit cycle in QCD.

1. CHIRAL LIMIT OF QCD AND EFFECTIVE FIELD THEORY

In QCD, the up, down and strange quarks are light compared to the typical hadronic scale. Therefore, it is a good first approximation to consider the $SU(3)$ chiral limit with $m_u = m_d = m_s = 0$ (and all heavy quarks decoupled, $m_c = m_b = m_t = \infty$). In that limit, QCD possesses an exact $SU(3)_L \times SU(3)_R$ chiral symmetry and has a high degree of symmetry because gluons are flavor-blind. However, the full symmetry is not shared by the vacuum, the chiral symmetry is spontaneously broken with the appearance of eight massless pseudoscalar mesons, to be identified with the pions, the kaons and the eta. These mesons are pseudo-Goldstone bosons because of the explicit symmetry breaking through the quark masses, collected in the quark mass matrix \mathcal{M} . The consequences of the explicit and the spontaneous symmetry breaking can be analyzed utilizing an Effective Field Theory (EFT), Chiral Perturbation Theory (CHPT), or variants thereof. For a review, see e.g. [1]. For the following discussion, I briefly recapitulate some salient features of the chiral limit (CL) of QCD. While S-matrix elements exist in the CL for arbitrary momenta, the approach to the CL is non-analytic in \mathcal{M} , which leads to the famous “chiral logs” as pointed out by many. It is important that there further exists the so-called decoupling theorem [2]: Leading chiral non-analytic terms stem from pion (Goldstone boson) one-loop graphs coupled to pions (Goldstone bosons) or nucleons (ground state baryons). This means that resonances like the ρ or the Δ decouple, and this constraint must be implemented when one constructs EFTs with explicit resonance degrees of freedom. Furthermore, the chiral limit of QCD and the large N_c limit do not commute, which is another constraint when setting up EFTs. More on that below. I will now discuss various issues related to the EFT of QCD that are pertinent to the structure of baryons.

2. QUARK MASS DEPENDENCE OF NUCLEON AND DELTA MASSES

CHPT can be used to connect the results of lattice simulations at unphysical quark masses with the world at their physical values. Such procedures are frequently denoted as “chiral extrapolations”. Of course, as the quark masses increase, the pion (and also the kaon) becomes heavier, eventually rendering such a procedure meaningless. More lattice results at lower quark masses are urgently called for, but it is very instructive to set up the framework and study how far one can get. Since this topic is still in its developing phase, I will not try to cover all possible angles but rather concentrate on some recent work concerning the nucleon, the delta and also the ground state octet baryons. For other views, the reader is referred to the contributions of Leinweber [3] and Procura [4] to this conference. The nucleon mass and general features of the nucleon were studied in [5] utilizing dimensional (DR) and cut-off (CR) regularization, working to fourth order in the chiral expansion,

$$m_N = m_0 - 4c_1 M_\pi^2 - \frac{3g_A^2 M_\pi^3}{32\pi F_\pi^2} + k_1 M_\pi^4 \ln \frac{M_\pi}{m_N} + k_2 M_\pi^4 + \mathcal{O}(M_\pi^5) , \quad (1)$$

with m_0 the nucleon mass in the *chiral* $SU(2)$ limit ($m_u = m_d = 0$, m_s fixed at its physical value), M_π (F_π) the pion mass (decay constant), and c_1, k_1, k_2 are (combinations of) second and fourth order LECs that can e.g. be determined in the CHPT analysis of pion-nucleon scattering, see [6]. With this input, one finds $m_0 = 880$ MeV and a large theoretical uncertainty for pion masses larger than ~ 550 MeV. This work was recently extended to the three-flavor case [7], cf. the left panel in Fig. 1. As shown in that figure, a slight readjustment of the fourth order LECs earlier determined in [8] allows quite well to describe the trend of the (partially quenched) data from the MILC collaboration [9], compare the dot-dashed (new LECs) and the dotted (old LECs) line. Also shown in that figure are the result at third order (dashed) and including an improvement term (solid line) as suggested in [5]. Clearly, one has to work at fourth order if one wants to describe the lattice data below pion masses of about 600 MeV (the curves are only shown for larger M_π to better exhibit the trends). The pion mass dependence of the nucleon mass is very similar to what is found in the two-flavor case. The kaon mass dependence of m_N is also studied [7]; it is less precisely determined. One finds e.g. for the nucleon mass in the *chiral* $SU(3)$ limit the wide range $m_0^{SU(3)} \in [710, 1070]$ MeV. In that paper, for the first time results for the Λ , the Σ and the Ξ were given and compared to the few existing lattice data. These results are encouraging and the full machinery of partially quenched CHPT (see e.g. [10]) should be applied to the MILC data. To make similar statements for the delta resonance, one has to include it in the EFT explicitly. This can be done by counting the $N\Delta$ mass splitting as an additional parameter - which, however, does not vanish in the chiral limit. This is a nice example of the decoupling theorem in the CL and the resulting non-commutativity with the large N_C limit. The generalized (phenomenological) power counting including $m_\Delta - m_N$ is called the “small scale expansion” (SSE) [11]. The quark mass dependence of the delta mass based on a covariant version of the SSE [12] was recently worked out and the central results of this study are shown in the right panel of Fig. 1. The effective $\pi N\Delta$ Lagrangian includes additional parameters that allow for a simultaneous description of the nucleon and the delta mass. It is interesting that the

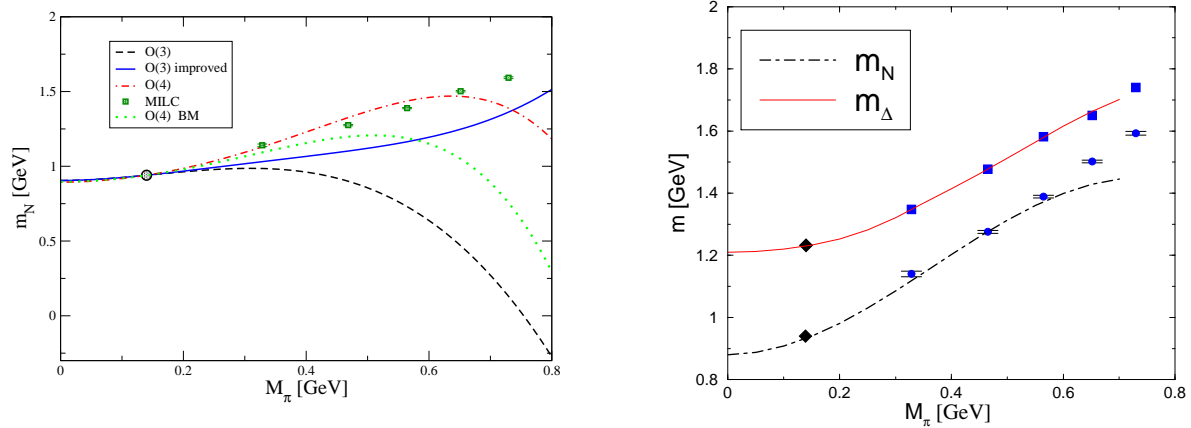


Figure 1. Left panel: Pion mass dependence of the nucleon mass in an SU(3) calculation at third (dashed), improved third (solid) and fourth (dot-dashed) order, respectively. The dotted line represents the fourth order calculation from [8]. Right panel: Pion mass dependence of m_N (dot-dashed) and m_Δ (solid line) based on a covariant calculation in the small scale expansion. The filled diamonds denote the physical values of m_N and m_Δ at $M_\pi = 140$ MeV. The data in both panels are from MILC [9].

lattice data shown in the figure (again from MILC) seem to indicate a stronger slope of $m_N(M_\pi)$ than for $m_\Delta(M_\pi)$ for pion masses slightly above the physical point. This indicates that the pion-delta sigma term is sizeably smaller than the pion-nucleon one - in the strict SU(6) limit one would expect $\sigma_{\pi\Delta} = \sigma_{\pi N}$. This indicates that pion cloud effects are less pronounced in the baryon resonances than in the ground-state. Note that the MILC data shown in the figure are again from partially quenched SU(3) simulations - thus it would be interesting to repeat this analysis in the framework presented in [13].

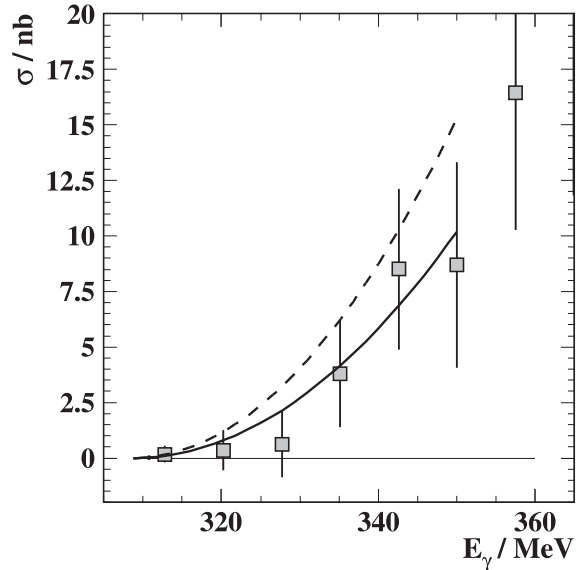
3. PHOTO-NUCLEON/NUCLEAR PROCESSES

Pion photo- and electroproduction has been established as one of the major testing grounds of baryon chiral dynamics. Shortly after the detailed investigations of single neutral pion production off nucleons (see e.g. the review [1]), electromagnetic two-pion production off protons and neutrons was also considered. Naively, one would estimate the cross section to be very small, more precisely $\sigma_{\text{tot}}(\gamma p \rightarrow \pi^0 \pi^0 p) \ll \sigma_{\text{tot}}(\gamma p \rightarrow \pi^+ \pi^- p)$ because of the “double Kroll-Rudermann suppression”. In the pioneering paper in 1994 [14] it was, however, shown, that a) there are large chiral loop corrections in the $2\pi^0$ channel and b) that the leading Δ -contributions cancel. At third order and in the threshold region, one finds indeed that $\sigma_{\text{tot}}(\gamma p \rightarrow \pi^0 \pi^0 p) > \sigma_{\text{tot}}(\gamma p \rightarrow \pi^+ \pi^- p)$. This was sharpened a few years later when the calculation was extended to fourth order and the cross section for $\gamma p \rightarrow \pi^0 \pi^0 p$ could be given in analytic form [15]:

$$\sigma_{\text{tot}}(E_\gamma) = \mathcal{C} [\text{nb}] \left(\frac{E_\gamma - E_\gamma^{\text{thr}}}{10 \text{ MeV}} \right)^2, \quad \mathcal{C} = \begin{cases} 0.6 & \text{central value,} \\ 0.9 & \text{upper limit,} \end{cases} \quad (2)$$

Here, $E_\gamma^{\text{thr}} = 308.8 \text{ MeV}$ is the threshold energy and the constant \mathcal{C} contains some low-energy constants, in particular one related to the decay of the Roper into two pions,

Figure 2. Total cross section for two-neutral-pion photo-production off the proton, $\gamma p \rightarrow \pi^0 \pi^0 p$, in the threshold region as a function of the photon energy E_γ . The solid line is the fourth order result of [15] and the dashed line refers to the upper limit also given in that paper. The data are from the recent measurement of the TAPS collaboration [17]. The threshold energy is $E_\gamma^{\text{thr}} = 308.8 \text{ MeV}$.



$N^*(1440) \rightarrow N(\pi\pi)_{\text{S-wave}}$. From the earlier study of the reaction $\pi N \rightarrow \pi\pi N$ in CHPT a central value as well as an upper limit for this constant could be given [16] which reflects itself in the values for \mathcal{C} in Eq. (2). This sharp prediction of baryon CHPT could only be tested many years later because the predicted cross section in the threshold region is very small and a dedicated experiment with the TAPS detector at MAMI had to be performed. The TAPS collaboration published their result in 2004 and it agrees beautifully with the central prediction, cf. Fig. 3. They even state that the upper limit for the $N^*(1440) \rightarrow N(\pi\pi)_{\text{S-wave}}$ can be excluded. The experimental result is also in agreement with a result obtained in the chiral unitary model of the Valencia group [18], where the pion loop effects were generated by pion rescattering in the scalar-isoscalar channel.

Pion photo- and electroproduction off the deuteron allows to test the counterintuitive CHPT prediction that the S-wave amplitude $E_{0+}(\pi^0 n)$ is larger in magnitude than the one for the proton, $E_{0+}(\pi^0 p)$ [19]. In a naive dipole picture, one expects $E_{0+}(\pi^0 n) = 0$. In the case of photoproduction, only one measurement of the total cross section with sufficient accuracy has ever been performed at the now defunct Saskatoon accelerator SAL [20]. The extrapolated threshold cross section agrees with the earlier CHPT prediction [21] and clearly rules out a vanishing neutron dipole amplitude. At that time, the hybrid approach was used, i.e. the deuteron wave functions were taken from precise phenomenological potentials and then applied to the kernel calculated within CHPT. More data have become available with the MAMI measurements of coherent pion electroproduction, $\gamma(Q^2)d \rightarrow \pi^0 d$, at the (negative of the) photon virtuality $Q^2 = 0.1 \text{ GeV}^2$ [22]. This reaction has been analyzed in detail in Refs.[23]. In particular, the threshold multipole expansion has been developed, consistent deuteron wave functions from next-to-next-to-leading order nuclear chiral EFT [24] were utilized and boost effects were considered. The pertinent matrix elements decompose into the so-called single scattering and the three-body terms. While the former contain the information on the elementary proton and the neutron amplitudes, the latter are parameter-free at third order. The fourth order three-body corrections

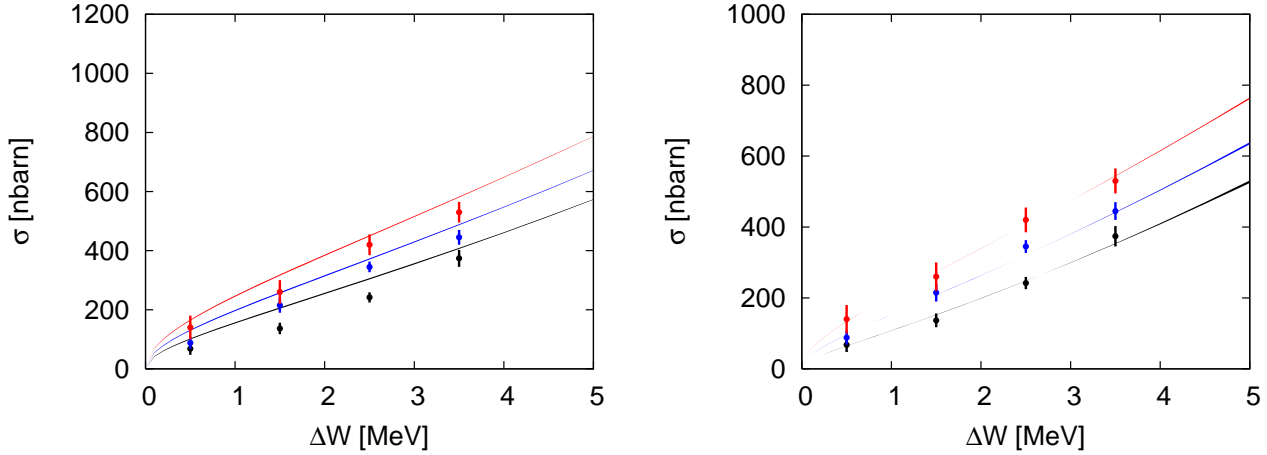


Figure 3. Left panel: Total cross section as a function of ΔW for three different values of the photon polarization in comparison to the MAMI data [22] for fit 1 and the NNLO wave functions. ΔW is the pion energy above threshold. The upper/middle/lower band corresponds to the largest/medium/smallest value of ε . Right panel: Same for fit 2.

contain two four-nucleon LECs. In the fits shown in Fig. 3, these were considered as free parameters. The two fits shown in that figure represent the theoretical uncertainty at that order. In fit 1 (left panel), one fits to the longitudinal deuteron S-wave multipole $|L_d|$ at $Q^2 = 0.1 \text{ GeV}^2$ as deduced in [22] whereas for fit 2 a best description of the total cross section data is achieved. One notices that the uncertainty due to the variation in the chiral wave functions is very small, the various lines are indeed bands that cover the set of wave functions given in [24]. Again, these data are clearly indicative of non-vanishing longitudinal and transverse neutron dipole amplitudes. Also, the resulting deuteron electric dipole amplitude at the photon point $Q^2 = 0$ is consistent with the SAL result [20].

The last topic I wish to address is the extension of these calculations to the delta region. For that, one must include the Δ as an active degree of freedom in the EFT. Of particular interest are the P-wave multipoles P_i ($i = 1, 2, 3$),

$$P_1 = 3E_{1+} + M_{1+} - M_{1-} , \quad P_2 = 3E_{1+} - M_{1+} + M_{1-} , \quad P_3 = 2M_{1+} - M_{1-} , \quad (3)$$

in terms of the more common electric (E_{1+}) and magnetic ($M_{1\pm}$) P-waves. At threshold, one can derive *low-energy theorems (LETs)* for the slope of P_1 and of P_2 , while P_3 is dominated by the delta [19]. These LETs were successfully tested [25]. In Fig. 4 I show the preliminary results of a second order study in the covariant SSE [26] in comparison to the second order covariant nucleon CHPT and the phenomenological MAID analysis [27]. The description of the multipole P_3 is already quite accurate at this order, consistent with the Δ -saturation of the corresponding third order LEC in the deltaless theory (which gives a vanishing P_3 at this order). For the other two multipoles loop and higher order tree effects are needed to obtain a precise description, again consistent with the expectations from heavy baryon CHPT [19]. Still, the second order covariant SSE calculation already captures the trend of the data (given by the MAID result).

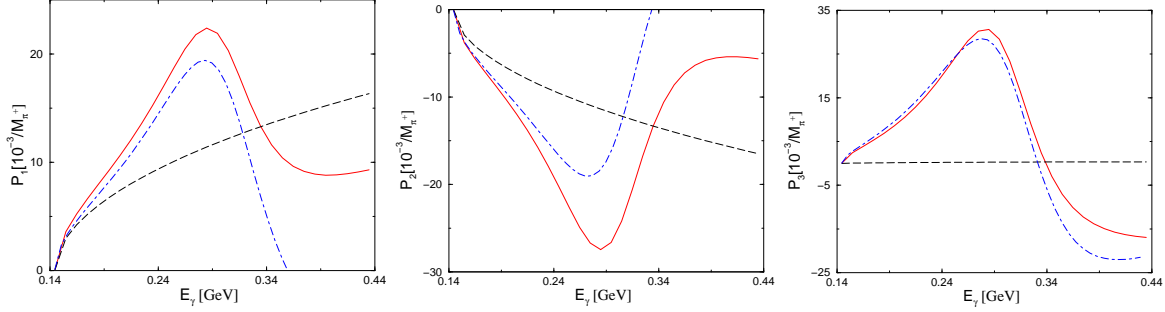


Figure 4. P-wave multipoles P_1 , P_2 and P_3 in $\gamma p \rightarrow \pi^0 p$ as a function of the photon energy E_γ . Solid lines: Second order calculation in the covariant SSE; dot-dashed lines: second order calculation in covariant CHPT (utilizing infrared regularization); dashed line: result of the MAID analysis.

4. QUARK MASS DEPENDENCE OF THE NUCLEAR FORCES

Because of the smallness of the up and down quark masses, one does not expect significant changes in systems of pions or pions and one nucleon when the quark masses are set to zero (with the exception of well understood chiral singularities like e.g. in the pion radius or the nucleon polarizabilities). The situation is more complicated for systems of two (or more) nucleons. Here, I report on some work [28] that is mostly concerned with the properties of the deuteron and the S-wave scattering lengths as a function of the quark (pion) mass. These questions are not only of academic interest, but also of practical use for interpolating results from lattice gauge theory. E.g. the S-wave scattering lengths have been calculated on the lattice using the quenched approximation [29]. Another interesting application is related to imposing bounds on the time-dependence of some fundamental coupling constants from the NN sector, as discussed in [30]. To address this issue, at NLO the following contributions have to be accounted for (in addition to the LO OPE and contact terms without derivatives): i) contact terms with two derivatives or one M_π^2 -insertion, ii) renormalization of the OPE, iii) renormalization of the contact terms, and iv) two-pion exchange (TPE). This induces *explicit* and *implicit* quark mass dependences. In the first category are the pion propagator that becomes Coulomb-like in the chiral limit or the M_π^2 corrections to the leading contact terms. These are parameterized by the LECs $\bar{D}_{S,T}$ at NLO. These LECs can at present only be estimated using dimensional analysis and resonance saturation [31]. The implicit pion mass dependence enters at NLO through the pion-nucleon coupling constant (note that the quark mass dependence of the nucleon mass only enters at NNLO) expressed through the pion mass dependence of g_A/F_π in terms of the quantity

$$\Delta = \left(\frac{g_A^2}{16\pi^2 F_\pi^2} - \frac{4}{g_A} \bar{d}_{16} + \frac{1}{16\pi^2 F_\pi^2} \bar{l}_4 \right) (M_\pi^2 - \tilde{M}_\pi^2) - \frac{g_A^2 \tilde{M}_\pi^2}{4\pi^2 F_\pi^2} \ln \frac{\tilde{M}_\pi}{M_\pi}. \quad (4)$$

Here \bar{l}_4 and \bar{d}_{16} are LECs related to pion and pion-nucleon interactions, and the value of the varying pion mass is denoted by \tilde{M}_π in order to distinguish it from the physical

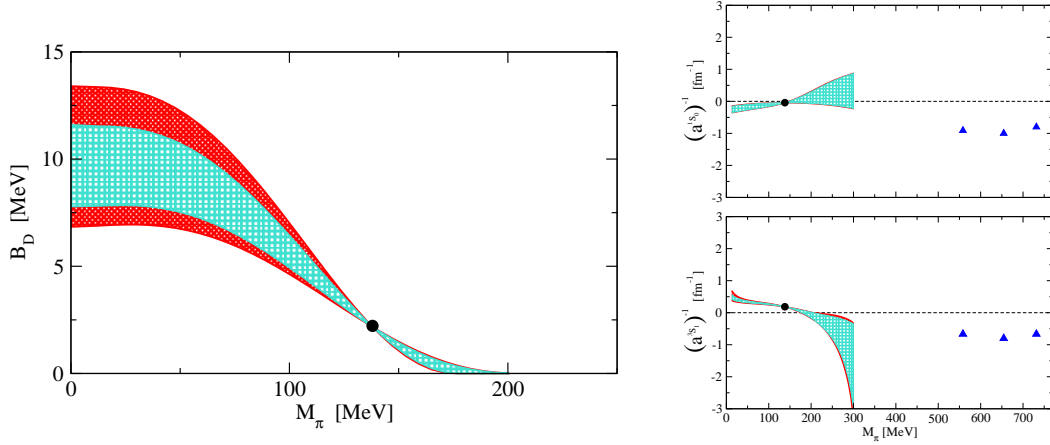


Figure 5. Left panel: Deuteron BE versus the pion mass. The shaded areas show allowed values. The light shaded band corresponds to our main result with the uncertainty due to the unknown LECs $\bar{D}_{S,T}$. The dark shaded band gives the additional uncertainty due to the uncertainty of \bar{d}_{16} . The heavy dot shows the BE for the physical case $\tilde{M}_\pi = M_\pi$. Right panel: The inverse S-wave scattering lengths as functions of \tilde{M}_π . The shaded areas represent the allowed values according to our analysis. The heavy dots corresponds to the values in the real world. The triangles refer to lattice QCD results from [29].

one denoted by M_π . In particular, \bar{d}_{16} has been determined in various fits to describe $\pi N \rightarrow \pi\pi N$ data, see [32]. The deuteron BE as a function of the pion mass is shown in Fig.5, we find that the deuteron is stronger bound in the chiral limit (CL) than in the real world, $B_D^{\text{CL}} = 9.6 \pm 1.9^{+1.8}_{-1.0}$ MeV, where the first indicated error refers to the uncertainty in the value of \bar{D}_{3S_1} and \bar{d}_{16} being set to its average value while the second indicated error shows the additional uncertainty due to the uncertainty in the determination of \bar{d}_{16} . We find no other bound states, although the higher $S = 1$ partial waves rise linear with momentum due to the Coulomb-like pion propagator. Last but not least, we found smaller (in magnitude) and more natural values for the two S-wave scattering lengths in the chiral limit, $a_{\text{CL}}(^1S_0) = -4.1 \pm 1.6^{+0.0}_{-0.4}$ fm, and $a_{\text{CL}}(^3S_1) = 1.5 \pm 0.4^{+0.2}_{-0.3}$ fm. As stressed in [28], one needs lattice data for pion masses below 300 MeV to perform a stable interpolation to the physical value of M_π , cf the right panel in Fig. 5. We conclude that nuclear physics in the chiral limit is much more natural than in the real world.

5. AN INFRARED LIMIT CYCLE IN QCD?

Another interesting application of the quark mass dependence of the nuclear forces is the recently conjectured infrared renormalization group limit cycle in the three-nucleon system [33]. A limit cycle is a non-trivial behaviour of a system under renormalization group (RG) transformations, more precisely a closed one-dimensional orbit under RG flow [34]. One of the signatures of such a limit cycle is discrete scale invariance, that is symmetry with respect to a scaling factor λ of the form λ^n , with n an integer. In the pionless effective field theory, i.e. the EFT with contact interactions only, an ultraviolet limit cycle was found for bosons with large scattering length [35] and for nucleons [36].

This EFT framework embodies the *Efimov effect*, namely that for systems with a very large S-wave two-particle scattering length, there is a large number of shallow 3-body bound states with the ratio of successive binding energies rapidly approaching a universal constant $\lambda_0^2 = e^{2\pi/s_0} \simeq 515$, with $s_0 = 1.00624$ a transcendental number [37]. For an excellent review of all the facets of such Efimov-type physics the reader is referred to [38]. In the nuclear physics case, the spin-singlet ($a(^1S_0)$) and the spin-triplet ($a(^3S_1)$) scattering lengths are both much larger than the range of the nuclear force $\sim 1/M_\pi$. Thus, one can describe few-nucleon systems based on point-like interactions with a leading order three-body force, that manifestly shows the asymptotic discrete scaling symmetry with the scaling factor $\lambda_0 = 22.7$ and the corresponding Efimov states. Because of the Efimov effect, the renormalization of the three-nucleon force is nontrivial [36] and involves an ultraviolet limit cycle. It is also interesting to consider the three-nucleon system at different quark masses. As can be seen from the right panel of Fig. 5, the deuteron becomes unbound at a critical value in the range $170 \text{ MeV} < M_\pi < 210 \text{ MeV}$. In the same range, $a(^1S_0)$ also diverges and the spin-singlet deuteron becomes bound for pion masses above 150 MeV. What does this imply e.g. for the triton? An exact limit cycle would require $1/a(^1S_0) = 1/a(^3S_1) = 0$. It was shown in [33] that for a pion mass of 175 MeV, the binding momentum of the pnn bound state is very small and an excited state of the triton appears (here, one has approximated the triton binding energy by its value at the physical pion mass). This can be considered as a hint that the system is close to a limit cycle. In fact, to leading order in QCD, one can only tune $M_\pi^2 \sim (m_u + m_d)$, so it was conjectured in Ref. [33] that by separately tuning m_u and m_d , one could make the singlet and the triplet scattering length diverge simultaneously. At this critical point, the deuteron and the spin-singlet deuteron would both have zero binding energy and the triton should have infinitely many shallow bound states, with their ratio rapidly approaching the constant $\lambda_0^2 \simeq 515$. It is a challenge for lattice gauge theory combined with EFT methods to indeed demonstrate the existence of such an infrared limit cycle in QCD.

6. BRIEF SUMMARY AND OUTLOOK

Let me briefly summarize. I have shown that baryon chiral perturbation theory has matured in the up and down quark sector, in particular, there exist now covariant formulations to include matter fields with spin-1/2 and spin-3/2 (in the latter case if one considers the $N\Delta$ mass splitting as a small parameter). These formulations do not render the heavy baryon formalism (HBCHPT) obsolete but rather extend and include it. In many cases, the computational simplicity of HBCHPT can and should still be used, but if one e.g. wants to make use of the analyticity properties by utilizing dispersion relations, relativistic formulations are the ones to be used, see e.g. the discussions in Refs. [39,40,41,42]. Chiral extrapolation functions based on CHPT with a *small (moderate)* theoretical uncertainty can be constructed for pion masses below 400 (550) MeV. Also, the extension to few-nucleon systems can be performed to a high accuracy, for a recent precise calculation in the two-nucleon system see [43]. May be the most tantalizing result is the conjecture of an infrared limit cycle in QCD as described above. I have discussed many applications of such schemes, ranging from the quark mass dependence of the nucleon and the delta mass to double neutral pion photoproduction off protons and

more. The extension to the strange quark sector certainly requires more work, making proper use of unitarization methods properly matched to CHPT amplitudes (for recent reviews, see [44,45]). Still, some results as the e.g. the SU(3) calculation of the quark mass dependence of the nucleon mass presented here are encouraging to further invest more effort in these topics.

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REFERENCES

1. V. Bernard, N. Kaiser and U.-G. Meißner, Int. J. Mod. Phys. E **4** (1995) 193 [arXiv:hep-ph/9501384].
2. J. Gasser and A. Zepeda, Nucl. Phys. B **174** (1980) 445.
3. D. Leinweber, *these proceedings*.
4. M. Procura, *these proceedings*.
5. V. Bernard, T. R. Hemmert and U.-G. Meißner, Nucl. Phys. A **732** (2004) 149 [arXiv:hep-ph/0307115].
6. P. Büttiker and U.-G. Meißner, Nucl. Phys. A **668** (2000) 97 [arXiv:hep-ph/9908247]; N. Fettes and U.-G. Meißner, Nucl. Phys. A **676** (2000) 311 [arXiv:hep-ph/0002162].
7. M. Frink and U.-G. Meißner, JHEP **0407** (2004) 028 [arXiv:hep-lat/0404018]; M. Frink, U.-G. Meißner and I. Scheller, *in preparation*.
8. B. Borasoy and U.-G. Meißner, Annals Phys. **254** (1997) 192 [arXiv:hep-ph/9607432].
9. C. W. Bernard *et al.*, Phys. Rev. D **64** (2001) 054506 [arXiv:hep-lat/0104002].
10. A. Walker-Loud, Nucl. Phys. A **747** (2005) 476 [arXiv:hep-lat/0405007].
11. T. R. Hemmert, B. R. Holstein and J. Kambor, J. Phys. G **24** (1998) 1831 [arXiv:hep-ph/9712496].
12. V. Bernard, T. R. Hemmert and U.-G. Meißner, Phys. Lett. B **565** (2003) 137 [arXiv:hep-ph/0303198]; and *forthcoming*.
13. B. C. Tiburzi and A. Walker-Loud, arXiv:hep-lat/0407030.
14. V. Bernard, N. Kaiser, U.-G. Meißner and A. Schmidt, Nucl. Phys. A **580** (1994) 475 [arXiv:nucl-th/9403013].
15. V. Bernard, N. Kaiser and U.-G. Meißner, Phys. Lett. B **382** (1996) 19 [arXiv:nucl-th/9604010].
16. V. Bernard, N. Kaiser and U.-G. Meißner, Nucl. Phys. B **457** (1995) 147 [arXiv:hep-ph/9507418].
17. M. Kotulla *et al.*, Phys. Lett. B **578** (2004) 63 [arXiv:nucl-ex/0310031].
18. L. Roca, E. Oset and M. J. Vicente Vacas, Phys. Lett. B **541** (2002) 77 [arXiv:nucl-th/0201054].

19. V. Bernard, N. Kaiser and U.-G. Meißner, Z. Phys. C **70** (1996) 483 [arXiv:hep-ph/9411287].
20. J. C. Bergstrom *et al.*, Phys. Rev. C **57** (1998) 3203.
21. S. R. Beane, V. Bernard, T. S. Lee, U.-G. Meißner and U. van Kolck, Nucl. Phys. A **618** (1997) 381 [arXiv:hep-ph/9702226].
22. I. Ewald *et al.*, Phys. Lett. B **499** (2001) 238 [arXiv:nucl-ex/0010008].
23. H. Krebs, V. Bernard and U.-G. Meißner, Nucl. Phys. A **713** (2003) 405 [arXiv:nucl-th/0207072]; Eur. Phys. J. A **22** (2004) 503 [arXiv:nucl-th/0405006].
24. E. Epelbaum, W. Glöckle and U.-G. Meißner, Eur. Phys. J. A **19** (2004) 401 [arXiv:nucl-th/0308010].
25. A. Schmidt *et al.*, Phys. Rev. Lett. **87** (2001) 232501 [arXiv:nucl-ex/0105010].
26. V. Bernard, T.R. Hemmert and U.-G. Meißner, *in preparation*.
27. see the website <http://www.kph.uni-mainz.de/MAID/>.
28. E. Epelbaum, Ulf-G. Meißner, and W. Glöckle, Nucl. Phys. A **714** (2003) 535 [arXiv:nucl-th/0207089]; nucl-th/0208040
29. M. Fukugita, Y. Kuramashi, M. Okawa, H. Mino and A. Ukawa, Phys. Rev. D **52** (1995) 3003 [arXiv:hep-lat/9501024].
30. S. R. Beane and M. J. Savage, Nucl. Phys. A **713**, 148 (2003) [arXiv:hep-ph/0206113].
31. E. Epelbaum, U.-G. Meißner, W. Glöckle and C. Elster, Phys. Rev. C **65** (2002) 044001 [arXiv:nucl-th/0106007].
32. N. Fettes, V. Bernard, and Ulf-G. Meißner, Nucl. Phys. A **699**, 269 (2000); N. Fettes, doctoral thesis, published in *Berichte des Forschungszentrum Jülich*, **3814**, (2000).
33. E. Braaten and H. W. Hammer, Phys. Rev. Lett. **91** (2003) 102002 [arXiv:nucl-th/0303038].
34. K. G. Wilson, Phys. Rev. D **3** (1971) 1818.
35. P. F. Bedaque, H. W. Hammer and U. van Kolck, Phys. Rev. Lett. **82** (1999) 463 [arXiv:nucl-th/9809025].
36. P. F. Bedaque, H. W. Hammer and U. van Kolck, Nucl. Phys. A **676** (2000) 357 [arXiv:nucl-th/9906032].
37. V. Efimov, Phys. Lett. **33B** (1970) 563.
38. E. Braaten and H. W. Hammer, arXiv:cond-mat/0410417.
39. U.-G. Meißner and J. A. Oller, Nucl. Phys. A **673** (2000) 311 [arXiv:nucl-th/9912026].
40. T. Becher and H. Leutwyler, JHEP **0106** (2001) 017 [arXiv:hep-ph/0103263].
41. B. Pasquini, D. Drechsel and L. Tiator, arXiv:nucl-th/0412038.
42. V. Pascalutsa, B. R. Holstein and M. Vanderhaeghen, Phys. Lett. B **600** (2004) 239 [arXiv:hep-ph/0407313].
43. E. Epelbaum, W. Glöckle and U.-G. Meißner, Nucl. Phys. A **747** (2005) 362 [arXiv:nucl-th/0405048].
44. U.-G. Meißner, AIP Conf. Proc. **717** (2004) 656 [arXiv:hep-ph/0309248].
45. B. Borasoy, arXiv:hep-ph/0402292.